

Parallel Gaussian Networks with a Common State-Cognitive Helper^{1 2}

Ruchen Duan,³ Yingbin Liang,³ Ashish Khisti,⁴ Shlomo Shamai (Shitz)⁵

Abstract

A class of state-dependent parallel networks with a common state-cognitive helper, in which K transmitters wish to send K messages to their corresponding receivers over K state-corrupted parallel channels, and a helper who knows the state information noncausally wishes to assist these receivers to cancel state interference. Furthermore, the helper also has its own message to be sent simultaneously to its corresponding receiver. Since the state information is known only to the helper, but not to the corresponding transmitters $1, \dots, K$, transmitter-side state cognition and receiver-side state interference are mismatched. Our focus is on the high state power regime, i.e., the state power goes to infinity. Three (sub)models are studied. Model I serves as a basic model, which consists of only one transmitter-receiver (with state corruption) pair in addition to a helper that assists the receiver to cancel state in addition to transmitting its own message. Model II consists of two transmitter-receiver pairs in addition to a helper, and only one receiver is interfered by a state sequence. Model III generalizes model I include multiple transmitter-receiver pairs with each receiver corrupted by independent state. For all models, inner and outer bounds on the capacity region are derived, and comparison of the two bounds leads to characterization of either full or partial boundary of the capacity region under various channel parameters.

1 Introduction

State-dependent network models have recently caught intensive attention. In these models, receivers are interfered by random state sequences, and some or all of the transmitters know the corresponding state sequences that interfere their targeted receivers noncausally, and exploit dirty paper coding to assist the receivers to cancel the state interference. For example,

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³Ruchen Duan and Yingbin Liang are with the Department of Electrical Engineering and Computer Science, Syracuse University, Syracuse, NY 13244 USA (email: {yliang06,rduan}@syr.edu).

⁴Ashish Khisti is with the Department of Electrical and Computer Engineering, University of Toronto, Toronto, ON, M5S3G4, Canada (email: akhisti@comm.utoronto.ca).

⁵Shlomo Shamai (Shitz) is with the Department of Electrical Engineering, Technion-Israel Institute of Technology, Technion city, Haifa 32000, Israel (email: sshlomo@ee.technion.ac.il).

the state-dependent broadcast channel has been studied in, e.g., [1, 2], the state-dependent multiple access channel (MAC) has been studied in, e.g., [3–5], the state-dependent relay channel has been studied in, e.g., [6, 7], and the state-dependent interference channel has been studied in, e.g., [8–11].

In this paper, we study a class of state-dependent parallel networks with a common state-cognitive helper (see Figure 1), in which K transmitters wish to send K messages to their corresponding receivers over K state-corrupted parallel channels, and a helper who knows the state information noncausally wishes to assist these receivers to cancel state interference. Furthermore, the helper also has its own message to be sent simultaneously to its corresponding receiver. Since the state information is known only to the helper, but not to the corresponding transmitters $1, \dots, K$, transmitter-side state cognition and receiver-side state interference are mismatched. Our goal is to investigate such a mismatched scenario in high state power regime, i.e., as the power of the state sequences go to infinity. This model is well justified in practical wireless networks. For example, in a cellular network, a base station likely causes interference to receivers in its adjacent cells, and such interference can be treated as state known at this base station. The base station can then serve as a helper to assist the receivers to cancel state interference, which is particularly desirable when the state power is large. Such a model suggests to exploit the state cognition for improving communication rates other than the traditional message cognition studied in the context of cognitive channels and networks.

This network model has a few properties that differentiate it from previous studies of state-dependent networks. In this model, the state knowledge is known only to the helper, which does not know the messages that it assists for transmission. This is different from the classic state-dependent channel in [12] and most of its followups, in which the transmitter knows both the message and the state. Although such a mismatch structure appeared also in some previously studied models such as the state-dependent multiple-access channel in [4], and the relay channel in [6, 7], the structure of multiple state-interfered receivers differentiates our model from these studies. Since our model has the nature of compound state interference at K receivers (i.e., the K receivers are corrupted by different states), the helper’s assistance scheme needs to trade off among the K receivers’ performances.

In this paper, we study three (sub)models of the state-dependent parallel networks with a common helper. Model I serves as a basic model, which consists of only one state-corrupted receiver ($K = 1$) and a helper that assists this receiver to cancel state interference in addition to transmitting its own message. Our study of this model provides necessary techniques to deal with state in the mismatched context for studying more complicated models II and III. In fact, this model can be viewed as the state-dependent Z -interference channel, in which the interference is only at receiver 1 caused by the helper. In contrast to the state-dependent Z -interference channel studied previously in [13], which assumes that state interference at both receivers are known to both (corresponding) transmitters, our model assumes that state interference is known noncausally only to the helper, not to the corresponding transmitter 1.

In general, it is challenging to design capacity-achieving schemes for such a system with

mismatched property. Clearly, it is not possible for transmitter 1 to directly cancel state interference due to the large state power. One natural idea is to apply lattice coding in high state power regime as in [14] for the state-dependent multiple access channel (MAC). However, lattice coding does not achieve the capacity for our model here. Another approach is to apply dirty paper coding [15]. However, the difficulty here lies in that the helper needs to resolve the tension between transmitting its own message and helping receiver 1 to cancel its interference.

In this paper, we design a layered coding scheme, in which a dirty paper coding scheme for the helper to assist state cancelation is superposed with the helper's transmission to its own message. Due to mismatched state cognition and interference, in our dirty paper coding scheme, correlation between the state variable and the state-cancelation variable is a design parameter, and can be chosen to optimize the rate region. This is in contrast to classical dirty paper coding [15], in which such a correlation parameter is fixed for fully canceling the state. Based on such a layered coding scheme, we derive achievable regions for both the discrete memoryless and Gaussian channels. We further derive an outer bound for the Gaussian channel in high state power regime. By comparing the inner and outer bounds, we characterize the boundary of the capacity region either fully or partially for all Gaussian channel parameters in high state power regime. Our result also implies that the capacity region is strictly inside the capacity region of the corresponding channel without state [16]. This is in contrast to the results for Costa type of dirty paper channels, for which dirty paper coding achieves the capacity of the corresponding channels without state.

We then further study model II, which consists of two transmitter-receiver pairs in addition to the helper, and only one receiver is interfered by a state sequence. Here, the challenge lies in the fact that the helper inevitably causes interference to receiver 2 while assisting receiver 1 to cancel the state. For this model, we start with the scenario with the helper fully assisting the receivers without transmitting its own message. We first derive an outer bound on the capacity region. We then develop a two-layer dirty paper coding scheme with one layer helping receiver 1 to cancel state via dirty paper coding, and with the other layer of dirty paper coding canceling the interference caused by the helper in assisting receiver 1. By comparing inner and outer bounds, we characterize two segments of the capacity region boundary. One segment corresponds to the case, in which our scheme achieves the point-to-point channel capacity for receiver 2 and certain positive rate for receiver 1. This implies that the helper is able to assist receiver 1 without causing interference to receiver 2 effectively. The other segment corresponds to the case, in which our scheme achieves the best single-user rate for receiver 1 with assistance of the helper, while receiver 2 treats the helper's signal as noise. Such a scheme is guaranteed by our outer bound to be the best to achieve the sum capacity under certain channel parameters. We further extend these results to the scenario with the helper sending its own message in addition to assisting the two receivers.

We finally study model III, in which a common helper assists multiple transmitter-receiver pairs with each receiver corrupted by an independently distributed state sequence. We note that this model is more general than model I, but does not include model II as a special case. This is because model III has each receiver (excluding the helper) being corrupted by an

infinitely powered state sequence, and hence never reduces to the model II, in which receiver 2 is not corrupted by a state sequence. This also leads to different technical challenges to characterize the capacity for model III due to the compound state interference. The same technical challenge is also reflected in the studies [14,17,18] of the state-dependent compound channel, for which the capacity is not known in general. As for model II, we also start with the scenario, in which the helper fully assists other users without sending its own message. We first derive a useful outer bound, which captures the sum rate limit due to the common helper. We then derive an inner bound based on a time-sharing scheme, in which the helper alternatively assists receivers. Somewhat interestingly, such a time-sharing scheme achieves the sum capacity under many channel parameters, although each individual transmitter may not be able to achieve its individual best rate. This is because these transmitters effectively have larger power during their transmissions in the time-sharing scheme so that the transmission rate matches the outer bound on the sum rate. We also characterize the full capacity region under certain channel parameters. We then extend our results to the general scenario with the helper also transmitting its own message.

The rest of the paper is organized as follows. In Section 2, we describe the channel model. In Sections 3, 4, and 5, we present our results for models I, II, and III, respectively. Finally, in Section 6, we conclude the paper with a few remarks.

2 Channel Model

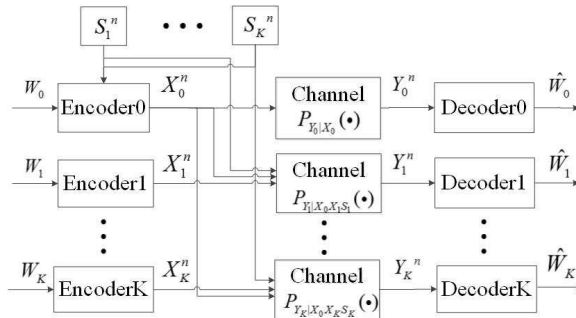


Figure 1: Parallel Gaussian channel model with a common state-cognitive helper.

In this paper, we investigate the state-dependent parallel network with a common state-cognitive helper (see Figure 1), in which K transmitters wish to send K messages to their corresponding receivers over state-corrupted parallel channels, and a helper who knows the state information noncausally wishes to assist these receivers to cancel state interference. Furthermore, the helper also has its own message to be sent simultaneously to its corresponding receiver.

More specifically, each transmitter (say transmitter k) has an encoder $f_k : \mathcal{W}_k \rightarrow \mathcal{X}_k^n$, which maps a message $w_k \in \mathcal{W}_k$ to a codeword $x_k^n \in \mathcal{X}_k^n$ for $k = 1, \dots, K$. The K inputs x_1^n, \dots, x_K^n are transmitted over K parallel channels, respectively. Each receiver (say receiver k) is interfered by an i.i.d. state sequence S_K^n for $k = 1, \dots, K$, which is unknown at none of

transmitters $1, \dots, K$ and receivers $1, \dots, K$. A common helper (referred to as transmitter 0) is assumed to know all state sequences S_K^n for $k = 1, \dots, K$ noncausally. Thus, the encoder at the helper, $f_0 : \mathcal{W}_0 \times \{\mathcal{S}_1^n, \dots, \mathcal{S}_K^n\} \rightarrow \mathcal{X}_0^n$, maps a message $w_0 \in \mathcal{W}_0$ and the state sequences $(s_1^n, \dots, s_K^n) \in \mathcal{S}_1^n \times \dots \times \mathcal{S}_K^n$ to a codeword $x_0^n \in \mathcal{X}_0^n$. The entire channel transition probability is given by $P_{Y_0|X_0} \prod_{k=1}^K P_{Y_k|X_0, X_k, S_k}$. There are $K = 1$ decoders with each at one receiver, $g_k : \mathcal{Y}_k^n \rightarrow \mathcal{W}_k$, maps a received sequence y_k^n into a message $\hat{w}_k \in \mathcal{W}_k$ for $k = 0, 1, \dots, K$.

Remark 1. *Without state interference, our model becomes the $K + 1$ -user Z -interference channel, in which the signal of transmitter 0 interferes all remaining K receivers.*

The average probability of error for a length- n code is defined as

$$P_e^{(n)} = \frac{1}{|\mathcal{W}_0| |\mathcal{W}_1| \dots |\mathcal{W}_K|} \sum_{w_0=1}^{|\mathcal{W}_0|} \sum_{w_1=1}^{|\mathcal{W}_1|} \dots \sum_{w_K=1}^{|\mathcal{W}_K|} Pr\{(\hat{w}_0, \hat{w}_1, \dots, \hat{w}_K) \neq (w_0, w_1, \dots, w_K)\}. \quad (1)$$

A rate tuple (R_0, R_1, \dots, R_K) is *achievable* if there exists a sequence of message sets $\mathcal{W}_k^{(n)}$ with $|\mathcal{W}_k^{(n)}| = 2^{nR_k}$ for $k = 0, 1, \dots, K$, and encoder-decoder tuples $(f_0^{(n)}, f_1^{(n)}, \dots, f_K^{(n)}, g_{0n}, g_{1n}, \dots, g_{Kn})$ such that the average error probability $P_e^{(n)} \rightarrow 0$ as $n \rightarrow \infty$. The *capacity region* is defined to be the closure of the set consists of all achievable rate pairs (R_0, R_1, \dots, R_K) .

In this paper, we study the following three Gaussian channel models.

In model I, $K = 1$. The channel outputs at receiver 0 and 1 for one symbol time are given by

$$Y_0 = X_0 + N_0, \quad (2a)$$

$$Y_1 = X_0 + X_1 + S_1 + N_1. \quad (2b)$$

In model II, $K = 2$, and the channel outputs at receivers 0, 1 and 2 for one symbol time are given by

$$Y_0 = X_0 + N_0, \quad (3a)$$

$$Y_1 = X_0 + X_1 + S_1 + N_1, \quad (3b)$$

$$Y_2 = X_0 + X_2 + N_2. \quad (3c)$$

$$(3d)$$

In model III, K is general and the channel outputs at receivers 0 and receivers $1, \dots, K$ for one symbol time are given by

$$Y_0 = X_0 + N_0, \quad (4a)$$

$$Y_k = X_0 + X_k + S_k + N_k, \quad k = 1, \dots, K \quad (4b)$$

In the above three models, the noise variables N_0, N_1, \dots, N_K and the state variable S_1, \dots, S_K are Gaussian distributed with distributions $N_0, \dots, N_K \sim \mathcal{N}(0, 1)$ and $S_k \sim \mathcal{N}(0, Q_k)$ for

$k = 1, \dots, K$, and all of the variables are independent and are i.i.d. over channel uses. The channel inputs X_0, X_1, \dots, X_K are subject to the average power constraints $\frac{1}{n} \sum_{i=1}^n X_{ki}^2 \leq P_k$ for $k = 0, 1, \dots, K$.

We are interested in the regime of high state power, i.e., as $Q_k \rightarrow \infty$ for $k = 1, \dots, K$. Our goal is to characterize the capacity region of the Gaussian channels in this regime.

3 Model I: $K = 1$

Model I with $K = 1$ is a basic model, in which the helper assists one transmitter-receiver pair. Understanding this model will help the study of the general parallel network. In this section, we first develop inner and outer bounds on the capacity region, and then characterize the boundary of the capacity region based on these bounds.

3.1 Inner Bound

The major challenge in designing an achievable scheme arises from the mismatched property due to transmitter-side state cognition and receiver-side state interference, i.e., state interference to receiver 1 is known noncausally only to transmitter 0 (the helper), not to the corresponding transmitter 1. Since we study the regime with large state power, transmitter 1 can send information to receiver 1 only if the helper assists to cancel the state. Thus, the helper needs to resolve the tension between transmitting its own message to receiver 0 and helping receiver 1 to cancel its interference. A simple scheme of time-sharing between the two transmitters in general is not optimal.

We design a layered coding scheme as follows. The helper splits its signal into two parts in a layered fashion: one (represented by X'_0 in Lemma 1) for transmitting its own message and the other (represented by U in Lemma 1) for helping receiver 1 to remove both state and signal interference. In particular, the second part of the scheme applies a single-bin dirty paper coding scheme, in which transmission of W_1 and treatment of state interference for decoding W_1 are performed separately by transmitters 1 and 0. This is because the helper knows the state but does not know the message (of transmitter 1) that the state interferes, and hence cannot encode this message via the regular multi-bin dirty paper coding as in [15]. Based on such a scheme, we obtain the following achievable rate region for the discrete memoryless channel, which is useful for deriving an inner bound for the Gaussian channel.

Lemma 1. *For the discrete memoryless model I, an inner bound on the capacity region consists of rate pairs (R_0, R_1) satisfying:*

$$R_0 \leq I(X'_0; Y_0) \tag{5a}$$

$$R_1 \leq I(X_1; Y_1 | U) \tag{5b}$$

$$R_1 \leq I(X_1 U; Y_1) - I(U; S_1 X'_0) \tag{5c}$$

for some distribution $P_{S_1} P_{X'_0} P_{U|S_1 X'_0} P_{X_0|U S_1 X'_0} P_{X_1} P_{Y_0 Y_1|S_1 X_0 X_1}$.

Proof. The proof is detailed in Appendix A. \square

Based on Lemma 1, we have the following simpler inner bound by adding a constraint to remove (5c) as a redundant bound.

Corollary 1. *For the discrete memoryless model I, an inner bound on the capacity region consists of rate pairs (R_0, R_1) satisfying:*

$$R_0 \leq I(X'_0; Y_0) \quad (6a)$$

$$R_1 \leq I(X_1; Y_1|U) \quad (6b)$$

for some distribution $P_{S_1}P_{X'_0}P_{U|S_1X'_0}P_{X_0|US_1X'_0}P_{X_1}P_{Y_0Y_1|S_1X_0X_1}$ that satisfies

$$I(U; Y_1) \geq I(U; S_1X'_0). \quad (7)$$

The inner bound in Corollary 1 corresponds to an intuitive achievable scheme based on successive cancellation. Namely, the condition guarantees that receiver 1 decodes the auxiliary random variable U first, and then removes it from its output and decodes the message, which results in the bound (6b). In particular, cancellation of U leads to cancellation of state interference at receiver 1.

We next derive an inner bound for the Gaussian channel of model I based on Corollary 1.

Proposition 1. *For the Gaussian channel of model I, an inner bound on the capacity region consists of rate pairs (R_0, R_1) satisfying:*

$$R_0 \leq \frac{1}{2} \log \left(1 + \frac{\bar{\beta}P_0}{\beta P_0 + 1} \right) \quad (8a)$$

$$R_1 \leq \frac{1}{2} \log \left(1 + \frac{P_1}{1 + (1 - \frac{1}{\alpha})^2 \beta P_0} \right) \quad (8b)$$

for some real constants $\alpha \geq 0$ and $0 \leq \beta \leq 1$ that satisfy

$$\alpha^2(\bar{\beta}P_0 + Q_1)(\beta P_0 + P_1 + 1) - 2\alpha\beta P_0(\bar{\beta}P_0 + Q_1) - \beta^2 P_0^2 \leq 0. \quad (9)$$

As $Q_1 \rightarrow \infty$, the preceding condition becomes $\alpha \leq \frac{2\beta P_0}{\beta P_0 + P_1 + 1}$.

Proof. Proposition 1 follows from Corollary 1 by choosing the joint Gaussian distribution for random variables as follows:

$$\begin{aligned} U &= X''_0 + \alpha(S_1 + X'_0), \quad X_0 = X'_0 + X''_0 \\ X'_0 &\sim \mathcal{N}(0, \beta P_0), \quad X''_0 \sim \mathcal{N}(0, \bar{\beta}P_0) \\ X_1 &\sim \mathcal{N}(0, P_1) \end{aligned}$$

where X'_0 , X''_0 , X_1 and S_1 are independent, $\alpha \geq 0$, $0 \leq \beta \leq 1$, and $\bar{\beta} = 1 - \beta$. \square

We note that in Proposition 1, the parameter α captures correlation between the state variable S and the auxiliary variable U for dealing with the state, and can be chosen to optimize the rate region. This is in contrast to the classical dirty paper coding [15], in which such correlation parameter is fixed for state cancelation. Therefore, although Corollary 1 may provide a smaller inner bound than that given in Lemma 1, it can be shown that two inner bounds are equivalent for our chosen auxiliary random variables and input distribution after optimizing over α .

3.2 Outer Bound

In this subsection, we provide an outer bound on the capacity region in high state power regime, i.e., as $Q \rightarrow \infty$.

Proposition 2. *For the Gaussian channel of model I, an outer bound on the capacity region for the regime when $Q_1 \rightarrow \infty$ consists of rate pairs (R_0, R_1) satisfying:*

$$R_1 \leq \frac{1}{2} \log(1 + P_1) \tag{10a}$$

$$R_0 + R_1 \leq \frac{1}{2} \log(1 + P_0). \tag{10b}$$

The bound (10a) on R_1 follows simply from the capacity of the point-to-point channel between transmitter 1 and receiver 1 without signal and state interference. The bound (10b) on the sum rate is limited only by the power P_0 of the helper, and does not depend on the power P_1 of transmitter 1. Intuitively, this is because P_0 is split for transmission of W_0 and for helping transmission of W_1 by removing state interference, and hence P_0 determines a trade-off between R_0 and R_1 . On the other hand, improving the power P_1 , although may improve R_1 , can also cause more interference for receiver 1 to decode the auxiliary variable for canceling state and interference. Thus, the balance of the two effects determine that P_1 does not affect the sum rate.

Proof. The proof is detailed in Appendix B. □

We further note that although the sum-rate upper bound (10b) can be achieved easily by keeping transmitter 1 silent (i.e., R_0 achieves the sum rate bound with $R_1 = 0$), we are interested in characterizing the capacity region (i.e., the trade-off between R_0 and R_1) rather than a single point that achieves the sum-rate capacity. In the next section, we characterize such optimal trade-off based on the sum-rate bound.

Remark 2. *The outer bound in Proposition 2 is strictly inside an achievable rate region of the corresponding channel without state interference (i.e., the Z-interference channel) [16]. This implies that the capacity region of our model is strictly inside that of the corresponding channel without state. This suggests that state interference does cause performance degradation for systems with mismatched state cognition and interference in high state power regime. This is in contrast to the results on Costa-type dirty paper channels [15], for which dirty paper coding achieves the capacity of the corresponding channels without state.*

3.3 Capacity Region

In this section, we characterize the boundary points of the capacity region for the Gaussian channel of model I based on the inner and outer bounds given in Propositions 1 and 2, respectively. We partition the Gaussian channel into three cases based on the conditions on the power constraints: (1) $P_1 \geq P_0 + 1$; (2) $P_0 - 1 \leq P_1 < P_0 + 1$ and (3) $0 \leq P_1 < P_0 - 1$. For each case, we optimize the dirty paper coding parameter α that satisfies $0 \leq \alpha \leq \frac{2\beta P_0}{\beta P_0 + P_1 + 1}$ to find achievable rate points that lie on the sum-rate upper bound (10b) in order to characterize the boundary points of the capacity region.

Case 1: $P_1 \geq P_0 + 1$. The capacity region is fully characterized as illustrated in Fig. 2.

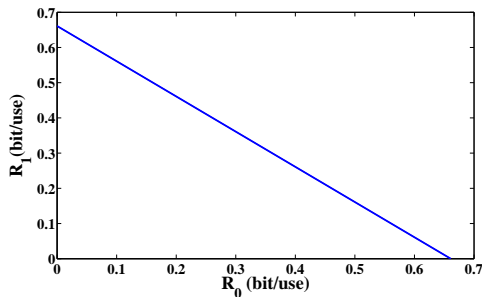


Figure 2: The capacity region for case 1 with $P_0 = 1.5$ and $P_1 = 3$.

Theorem 1. *For the Gaussian channel of model I in the regime when $Q_1 \rightarrow \infty$, if $P_1 \geq P_0 + 1$, the capacity region consists of the rate pairs (R_0, R_1) satisfying*

$$R_0 + R_1 \leq \frac{1}{2} \log(1 + P_0). \quad (11)$$

Proof. Let \tilde{P}_1 be the actual transmission power for transmitting W_1 . Then the inner bound (8b) on R_1 is optimized when $\alpha = \frac{2\beta P_0}{\beta P_0 + \tilde{P}_1 + 1}$. By setting $\tilde{P}_1 = \beta P_0 + 1$, the inner bound given in Proposition 1 matches the outer bound given in Proposition 2, and hence is the capacity region. \square

Theorem 1 implies that when P_1 is large enough, the power of the helper limits the system performance. Furthermore, since P_1 for transmission of W_1 causes interference to receiver 1 to decode the auxiliary variable for state and interference cancelation, beyond a certain value, increasing P_1 does not improve the rate region any more. Theorem 1 also suggests that in order to achieve different points on the boundary of the capacity region (captured by parameters β), different amounts of power \tilde{P}_1 should be applied.

Case 2: $P_0 - 1 \leq P_1 < P_0 + 1$. For this case, if $P_1 \geq 1$, i.e., P_1 is larger than the noise power, inner and outer bounds match over the line between the points A and B as illustrated in Fig. 3 (a) and (c), and thus optimal trade-off between R_0 and R_1 is achieved over the points on this line. If $P_1 < 1$, the inner and outer bounds match only at the rate point A as illustrated in Fig. 3 (b) and (d), which achieves the sum-rate capacity. We note that Fig. 3

(a) and Fig. 3 (b) are different from Fig. 3 (c) and Fig. 3 (d) in the outer bound. Fig. 3 (c) and Fig. 3 (d) correspond to the case with $P_0 \geq P_1$, and hence the capacity region is also upper bounded by the point-to-point capacity of R_1 . Such a bound is redundant in Fig. 3 (a) and Fig. 3 (b) which correspond to the case with $P_0 < P_1$, because P_0 is not large enough to perfectly cancel state and signal interference at receiver 1. However, in case 3, we show that this point-to-point capacity of R_1 is achievable simultaneously with a certain positive R_0 . We summarize the above capacity result in the following theorem.

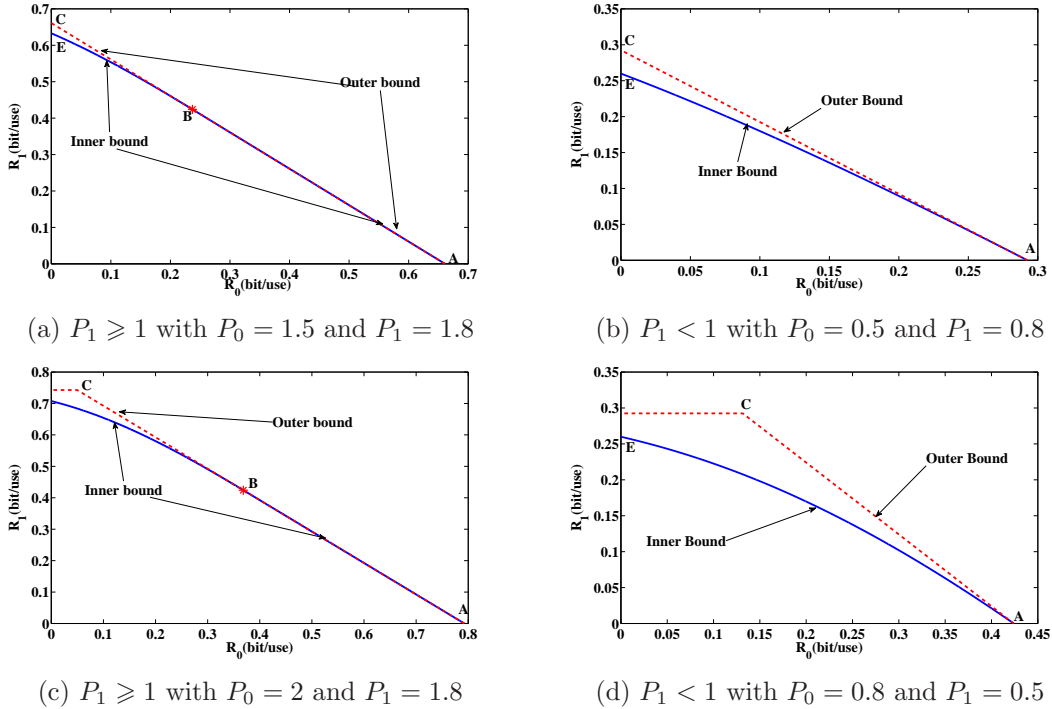


Figure 3: Inner and outer bounds for case 2, which match partially on the boundaries

Theorem 2. For the Gaussian channel of model I in the regime when $Q_1 \rightarrow \infty$, if $P_0 - 1 \leq P_1 < P_0 + 1$ and $P_1 \geq 1$, the rate points (R_0, R_1) on the line between $(\frac{1}{2} \log(1 + P_0), 0)$ (i.e., point A in Fig. 3 (b) and (d)) and $(\frac{1}{2} \log(1 + \frac{P_0 - P_1 + 1}{P_1}), \frac{1}{2} \log P_1)$ (i.e., point B in Fig. 3 (b) and (d)) are on the boundary of the capacity region.

Proof. For the case $P_0 - 1 \leq P_1 < P_0 + 1$, we also set $\alpha = \frac{2\beta P_0}{\beta P_0 + P_1 + 1}$. Plug α into (8b) and we have

$$R_0 \leq \frac{1}{2} \log \left(1 + \frac{\bar{\beta} P_0}{\beta P_0 + 1} \right) \quad (12a)$$

$$R_1 \leq \frac{1}{2} \log \left(1 + \frac{4\beta P_0 P_1}{4\beta P_0 + (P_1 + 1 - \beta P_0)^2} \right) \quad (12b)$$

When $P_1 \geq 1$, by setting $\beta = \frac{P_1-1}{P_0}$, we have an achievable rate point (R_0, R_1) given by $\left(\frac{1}{2} \log\left(1 + \frac{P_0-P_1+1}{P_1}\right), \frac{1}{2} \log(P_1)\right)$, which is point B in Figure 3. It can be seen that point B is also on the outer bound. Obviously, Point A is achievable for any set of P_0 and P_1 . Thus, by time sharing the line $A - B$ is on the boundary of the capacity region. \square

Case 3: $P_1 < P_0 - 1$. Similar to cases 2, the inner and outer bounds match partially over the sum rate bound, i.e., the two bounds match over the line between points A and B (see Fig. 4 (a)) if $P_1 \geq 1$ and match at only the point A (see Fig. 4 (b)) if $P_1 < 1$. However, differently from case 2, the inner and outer bounds also match when $R_1 = \frac{1}{2} \log(1 + P_1)$ over the line between points D and E (see Fig. 4 (a) and (b)). This is because the power P_0 of transmitter 0 in this case is large enough to fully cancel state and signal interference so that transmitter 1 is able to reach its maximum point-to-point rate to receiver 1 without interference. Furthermore, transmitter 0 is also able to simultaneously transmit its own message at a certain positive rate as reflected by the line $D-E$ in Fig. 4 (a) and (b). We summarize these results on the boundaries of the capacity region in the following theorem.

Theorem 3. *For the Gaussian channel of model I in the regime when $Q_1 \rightarrow \infty$, if $P_1 < P_0 - 1$ and $P_1 \geq 1$, then the points on the line between $\left(\frac{1}{2} \log(1 + P_0), 0\right)$ (i.e., point A in Fig. 4 (a)) and $\left(\frac{1}{2} \log\left(1 + \frac{P_0-P_1+1}{P_1}\right), \frac{1}{2} \log P_1\right)$ (i.e., point B in Fig. 4 (a)), and the points on the line between $\left(\frac{1}{2} \log\left(\frac{P_0+1}{P_1+2}\right), \frac{1}{2} \log(1 + P_1)\right)$ (i.e., point D in Fig. 4 (a)) and $\left(0, \frac{1}{2} \log(1 + P_1)\right)$ (i.e., point E in Fig. 4 (a)) are on the boundary of the capacity region.*

If $P_1 < P_0 - 1$ and $P_1 < 1$, then the point $\left(\frac{1}{2} \log(1 + P_0), 0\right)$ (i.e., point A in Fig. 4 (b)) and the points on the line between $\left(\frac{1}{2} \log\left(\frac{P_0+1}{P_1+2}\right), \frac{1}{2} \log(1 + P_1)\right)$ (i.e., point D in Fig. 4 (b)) and $\left(0, \frac{1}{2} \log(1 + P_1)\right)$ (i.e., point E in Fig. 4 (b)) are on the boundary of the capacity region.

Proof. For the case $P_1 < P_0 - 1$, the inner bound is equivalent to

Part I

$$0 \leq \beta \leq \frac{P_1 + 1}{P_0} \quad (13a)$$

$$R_0 \leq \frac{1}{2} \log \left(1 + \frac{\bar{\beta} P_0}{1 + \beta P_0} \right) \quad (13b)$$

$$R_1 \leq \frac{1}{2} \log(1 + \beta P_0) \quad (13c)$$

and Part II

$$\frac{P_1 + 1}{P_0} \leq \beta \leq 1 \quad (14a)$$

$$R_0 \leq \frac{1}{2} \log \left(1 + \frac{\bar{\beta} P_0}{1 + \beta P_0} \right) \quad (14b)$$

$$R_1 \leq \frac{1}{2} \log(1 + P_1) \quad (14c)$$

For Part I, when $P_1 \geq 1$, same as in case 2, the line $A - B$ is on the boundary of the capacity region, achievable region and outer bound is shown in Figure 4 (b). When $P_1 < 1$, only point A on the capacity boundary is obtained, the inner bound and outer bound are indicated in Figure 4. For part II, because the bound for α is larger than 1, we set $\alpha = 1$, which means that both interference and state can be fully canceled at receiver 1, and the transmitting rate for W_1 achieves the point-to-point channel capacity (as in (14c)). Here, by setting $\beta = \frac{P_1+1}{P_0}$, point D is achieved. Thus, the line $D - E$ is on the boundary of the capacity region. It is reasonable. Because for this case, P_0 is big enough to cancel the state for receiver 1 on one hand and cancel the interference caused by X_0 on the other hand. \square

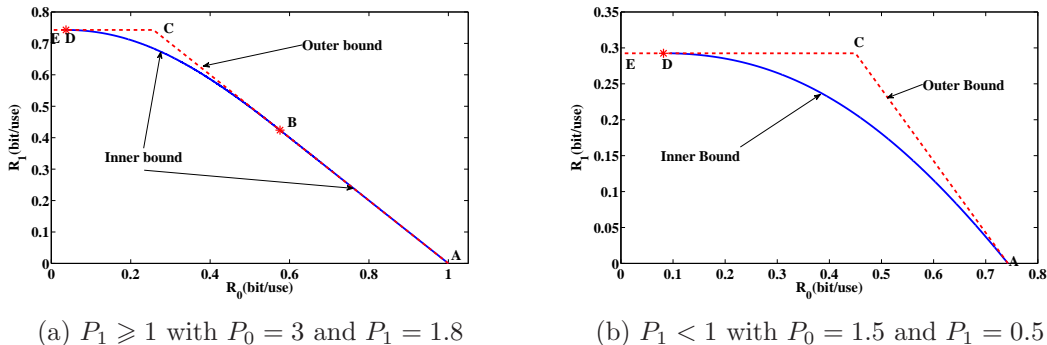


Figure 4: Inner and outer bounds for case 3, which match partially on the boundaries

4 Model II: $K=2$

In this section, we consider the Gaussian channel of model II with $K = 2$, but only receiver 1 interfered by an infinite state. We first study the scenario, in which the helper devotes to help two users without transmitting its own message, i.e., $W_0 = \phi$. We then extend the result to the more general scenario, in which the helper also has its own message destined for the corresponding receiver in addition to helping the two users, i.e., $W_0 \neq \phi$.

4.1 Scenario with Dedicated Helper ($W_0 = \phi$)

In this subsection, we study the scenario in which only receiver 1 is corrupted by a state sequence, and the helper (without transmission of its own messages) fully assists to cancel such state interference. Here, the challenge lies in the fact that the helper needs to assist receiver 1 to remove the state interference, but such signal inevitable causes interference to receiver 2. In this subsection, we first derive an outer bound, and then derive an inner bound based on the helper using a developed layered coding scheme with one layer assisting the state-interfered receiver, and the other layer canceling the interference in order to address the

challenge mentioned above. We then characterize the sum capacity under certain channel parameters and segments of the capacity boundary.

We first derive a useful outer bound for Model II.

Proposition 3. *For the Gaussian channel of model II with $W_0 = \phi$, an outer bound on the capacity region for the regime when $Q \rightarrow \infty$ consists of rate pairs (R_1, R_2) satisfying:*

$$R_1 \leq \min \left\{ \frac{1}{2} \log(1 + P_0), \frac{1}{2} \log(1 + P_1) \right\} \quad (15a)$$

$$R_2 \leq \frac{1}{2} \log(1 + P_2) \quad (15b)$$

$$R_1 + R_2 \leq \frac{1}{2} \log(1 + P_0 + P_2) \quad (15c)$$

Proof. The proof is relegated to Appendix C □

We note that (15a) represents the best single-user rate of receiver 1 with the helper dedicated to help it, as shown in Proposition 2, (15b) is the point-to-point capacity for receiver 2, and (15c) implies that although the two transmitters communicate over the parallel channel to their corresponding receivers, due to the shared common helper, the sum rate is still subject to a certain rate limit.

We next describe our idea to design an achievable scheme. We first note that although receiver 2 is not interfered by the state, the signal that the helper sends to assist receiver 1 to deal with the state still causes unavoidable interference to receiver 2. A natural idea to optimize the transmission rate to receiver 2 is simply to keep the helper silent. In this case, without the helper's assistance, receiver 1 gets zero rate due to infinite state power. In this paper, we design a novel scheme, which enables the point-to-point channel capacity for receiver 2 and certain positive rate for receiver 1 simultaneously; i.e., the helper is able to assist receiver 1 without causing interference to receiver 2. In our achievable scheme, the signal of the helper is split into two parts, represented by U and V as in Proposition 4. Here, U is designed to help receiver 1 to cancel the state while treating V as noise, and V is designed to help receiver 2 to cancel the interference caused by U . Since there is no state interference at receiver 2, U is decoded only at receiver 1. Based on such an achievable scheme, we obtain the following achievable region.

Proposition 4. *For the discrete memoryless model II with $W_0 = \phi$, an achievable region consists of the rate pair (R_1, R_2) satisfying*

$$R_1 \leq I(X_1; Y_1 U) \quad (16a)$$

$$R_1 \leq I(X_1 U; Y_1) - I(S_1; U) \quad (16b)$$

$$R_2 \leq I(X_2; Y_2 V) \quad (16c)$$

$$R_2 \leq I(X_2 V; Y_2) - I(V; U S_1) \quad (16d)$$

for some distribution $P_{S_1 U V X_0 X_1 X_2} = P_{S_1} P_{U V X_0 | S_1} P_{X_1} P_{X_2}$, where U and V are auxiliary random variables.

Proof. The proof is detailed in Appendix D. □

An straight-forward subregion for the inner bound is as follows.

Corollary 2. *For the discrete memoryless model II with $W_0 = \phi$, an achievable region consists of the rate pair (R_1, R_2) satisfying*

$$R_1 \leq I(X_1; Y_1 | U), \quad (17a)$$

$$R_2 \leq I(X_2; Y_2 | V), \quad (17b)$$

for some distribution $P_{S_1 U V X_0 X_1 X_2} = P_{S_1} P_{U V X_0 | S_1} P_{X_1} P_{X_2}$, where U and V are auxiliary random variables such that

$$I(U; Y_1) \geq I(U; S_1), \quad (18a)$$

$$I(V; Y_2) \geq I(V; U S_1). \quad (18b)$$

Following from the above achievable region, we obtain an achievable region for the Gaussian channel by setting an appropriate joint input distribution.

Proposition 5. *For the Gaussian channel of model II with $W_0 = \phi$, an inner bound on the capacity region for the regime when $Q_1 \rightarrow \infty$ consists of rate pairs (R_1, R_2) satisfying:*

$$R_1 \leq \frac{1}{2} \log \left(1 + \frac{P_1}{(1 - \frac{1}{\alpha})^2 P_{00} + P_{01} + 1} \right) \quad (19a)$$

$$R_2 \leq \frac{1}{2} \log \left(1 + \frac{P_2}{1 + \frac{(\beta-1)^2 P_{01} P_{00}}{P_{01} + \beta^2 P_{00}}} \right) \quad (19b)$$

where $P_{00} + P_{01} \leq P_0$, $P_{00}, P_{01} \geq 0$, $0 \leq \alpha \leq \frac{2P_{00}}{1+P_0+P_1}$, and $P_{01}^2 + 2\beta P_{00} P_{01} \geq \beta^2 P_{00} (P_{01} + P_2 + 1)$.

Proof. The achievability follows from Proposition 2 by choosing jointly Gaussian distribution for random variables as follows:

$$\begin{aligned} U &= X_{00} + \alpha S_1, \quad V = X_{01} + \beta X_{00} \\ X_{00} &\sim \mathcal{N}(0, P_{00}), \quad X_{01} \sim \mathcal{N}(0, P_{01}) \\ X_1 &\sim \mathcal{N}(0, P_1), \quad X_2 \sim \mathcal{N}(0, P_2) \end{aligned}$$

where X_{00}, X_{01}, X_1, X_2 and S_1 are independent. □

Comparing the inner and outer bounds given in Propositions 5 and 3, respectively, we characterize two segments of the boundary of the capacity region, over which the two bounds meet.

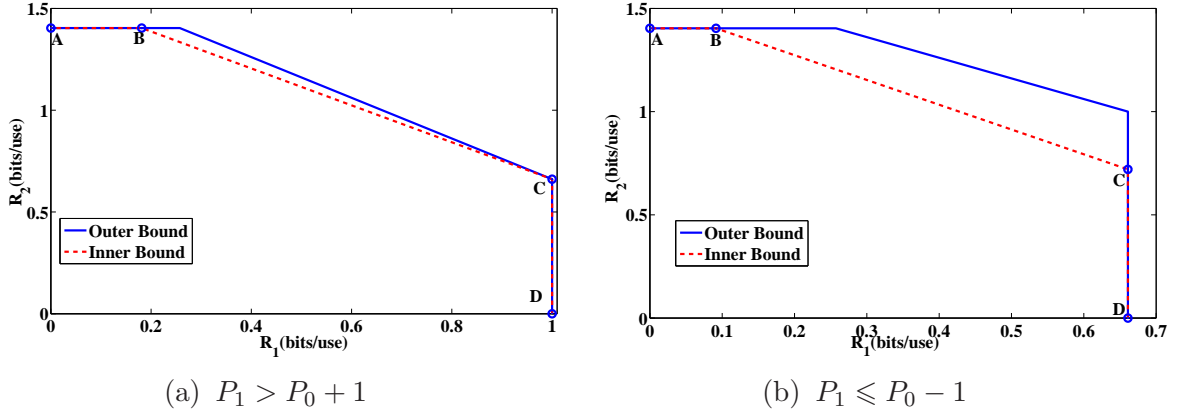


Figure 5: An illustration of the segments of the capacity boundary for the Gaussian channel of model II

Theorem 4. For the Gaussian channel of model II with $W_0 = \phi$, in the regime with $Q_1 \rightarrow \infty$, the line A-B (see Figure 5) is on the boundary of the capacity region. More specifically, if $\frac{1}{2}(1 + P_0 + P_1) \geq \frac{P_0^2}{P_0 + P_2 + 1}$, the line A-B is characterized as

$$\text{Point A : } \left(0, \frac{1}{2} \log(1 + P_2) \right)$$

$$\text{Point B : } \left(\frac{1}{2} \log \left(1 + \frac{4P_1 P_0^2}{(1 + P_0 + P_1)^2 (1 + P_0 + P_2) - 4P_1 P_0^2} \right), \frac{1}{2} \log(1 + P_2) \right)$$

If $\frac{1}{2}(1 + P_0 + P_1) < \frac{P_0^2}{P_0 + P_2 + 1}$, the line A-B is characterized as

$$\text{Point A : } \left(0, \frac{1}{2} \log(1 + P_2) \right)$$

$$\text{Point B : } \left(\frac{1}{2} \log \left(1 + \frac{P_1(P_0 + P_2 + 1)}{P_0 + (P_0 + 1)(P_2 + 1)} \right), \frac{1}{2} \log(1 + P_2) \right)$$

Furthermore, the line C-D (see Figure 5) is also on the boundary of the capacity region. If $P_1 > P_0 + 1$, the line C-D is characterized as

$$\text{Point C : } \left(\frac{1}{2} \log(1 + P_0), \frac{1}{2} \log \left(1 + \frac{P_2}{P_0 + 1} \right) \right)$$

$$\text{Point D : } \left(\frac{1}{2} \log(1 + P_0), 0 \right)$$

as illustrated in Figure 5-(a).

If $P_1 \leq P_0 - 1$, the line C-D is characterized as

$$\text{Point C : } \left(\frac{1}{2} \log(1 + P_1), \frac{1}{2} \log \left(1 + \frac{P_2}{P_1 + 2} \right) \right)$$

$$\text{Point D : } \left(\frac{1}{2} \log(1 + P_1), 0 \right)$$

as illustrated in Figure 5-(b).

Proof of Theorem 4. We first show that the line A - B is achievable. The point A is achievable by keeping the helper silent. To show that the point B is achievable, we set $\beta = 1$ in Proposition 5, and hence the achievable rate R_2 in (19b) reaches the point-to-point channel capacity, and the condition $P_{01}^2 + 2\beta P_{00}P_{01} \geq \beta^2 P_{00}(P_{01} + P_2 + 1)$ becomes $P_{00} \leq \frac{P_0^2}{P_0 + P_2 + 1}$.

When $\frac{1}{2}(1 + P_0 + P_1) \geq \frac{P_0^2}{P_0 + P_2 + 1}$, we have $\alpha \leq \frac{2P_{00}}{1 + P_0 + P_1} \leq \frac{2P_0^2}{(1 + P_0 + P_1)(P_0 + P_2 + 1)} \leq 1$. Thus, setting $\alpha = \frac{2P_{00}}{1 + P_0 + P_1}$ and $P_{00} = \frac{P_0^2}{P_0 + P_2 + 1}$, the point $\left(\frac{1}{2} \log\left(\frac{(1 + P_0 + P_1)^2(1 + P_0 + P_2)}{(1 + P_0 + P_1)^2(1 + P_0 + P_2) - 4P_0^2}\right), \frac{1}{2} \log(1 + P_2)\right)$ is achieved.

When $\frac{1}{2}(1 + P_0 + P_1) \leq \frac{P_0^2}{P_0 + P_2 + 1}$, we have $\alpha \leq \frac{2P_{00}}{1 + P_0 + P_1} \leq \frac{2P_0^2}{(1 + P_0 + P_1)(P_0 + P_2 + 1)} \geq 1$. By setting $\alpha = 1$ and $P_{00} = \frac{P_0^2}{P_0 + P_2 + 1}$, the point $\left(1 + \frac{P_1(P_0 + P_2 + 1)}{P_0 + (P_0 + 1)(P_2 + 1)}, \frac{1}{2} \log(1 + P_2)\right)$ is achieved.

The line C - D on the capacity boundary corresponds with the case when the helper is dedicated to assist receiver 1, and receiver 2 treats X_0 as noise, i.e., $\beta = 0$ in Proposition 5.

According to the result in Theorem 1 and Theorem 2, the only possible cases when the achievable rate for R_1 matches the outer bound is when $P_1 \leq P_0 - 1$ and $P_1 \geq P_0 + 1$.

When $P_1 \leq P_0 - 1$, by setting $\alpha = 1$ and $P_{00} = \tilde{P}_0 = P_1 + 1$, the rate point $\left(\frac{1}{2} \log(1 + P_1), \frac{1}{2} \log\left(1 + \frac{P_2}{P_1 + 2}\right)\right)$ is achieved, which is point C in Figure 5 (a). Particularly, the actual power used for the helper is $P_1 + 1$ rather than P_0 , because larger P_0 does not better help decoding R_1 and increase the interference to receiver 2. It is obvious that point D is achievable and points C and D are also on the outer bound. Hence, the points on the line C - D is on the capacity boundary.

When $P_1 \geq P_0 + 1$, by setting the actual power of transmitter 1 as $\tilde{P}_1 = P_0 + 1$, $P_{00} = P_0$ and $\alpha = \frac{P_0}{1 + P_0}$, the rate point $\left(\frac{1}{2} \log(1 + P_0), \frac{1}{2} \log\left(1 + \frac{P_2}{1 + P_0}\right)\right)$ is achieved, which is point C in Figure 5 (b). This point also achieves the sum capacity for the channel in model II. It is obvious that point D is achievable and points C and D are also on the outer bound. Hence, the points on the line C - D is on the capacity boundary. \square

The capacity result for the line A - B in Theorem 4 indicates that our coding scheme effectively enables the helper to assist receiver 1 without causing interference to receiver 2. Hence, R_2 achieves the corresponding point-to-point channel capacity, while transmitter 1 and receiver 1 communicate at a certain rate R_1 with the assistance of the helper.

The capacity result for the line C - D in Theorem 4 can be achieved based on a scheme, in which the helper assists receiver 1 to deal with the state and receiver 2 treats the helper's signal as noise. Such a scheme is guaranteed to be the best by the outer bound if receiver 1's rate is maximized.

Theorem 4 implies that if $P_1 \geq P_0 + 1$, the sum capacity is achieved by the point C as illustrated in Figure 5-(a).

Corollary 3. *For the Gaussian channel of model II with $W_0 = \phi$, in the regime with $Q_1 \rightarrow \infty$, if $P_1 \geq P_0 + 1$, the sum capacity is given by $\frac{1}{2} \log(1 + P_0 + P_2)$.*

4.2 Extension: $W_0 \neq \phi$

In this subsection, we study the scenario, in which the helper also has its own message to transmit in addition to assisting the state-corrupted receivers, i.e., $W_0 \neq \phi$. The results we present below extend those in the preceding subsection for the scenario with $W_0 = \phi$. The proof techniques are similar and hence are omitted.

We first provide an outer bound for the Gaussian channel, which is generalization of Propositions 2 and 3.

Proposition 6. *For the Gaussian channel of model II, an outer bound on the capacity region for the regime when $Q_1 \rightarrow \infty$ consists of rate pairs (R_1, R_2) satisfying:*

$$R_0 \leq \frac{1}{2} \log(1 + P_0) \quad (24a)$$

$$R_1 \leq \min \left\{ \frac{1}{2} \log(1 + P_0), \frac{1}{2} \log(1 + P_1) \right\} \quad (24b)$$

$$R_2 \leq \frac{1}{2} \log(1 + P_2) \quad (24c)$$

$$R_0 + R_1 + R_2 \leq \frac{1}{2} \log(1 + P_0 + P_2) \quad (24d)$$

By following similar steps as in Proposition 4 and Corollary 2, we derive an achievable region for the discrete memoryless model II as follows.

Proposition 7. *For the discrete memoryless model II, an achievable region consists of the rate tuple (R_0, R_1, R_2) satisfying*

$$R_0 \leq I(X_{00}; Y_0) \quad (25a)$$

$$R_1 \leq I(X_1; Y_1|U) \quad (25b)$$

$$R_2 \leq I(X_2; Y_2|V) \quad (25c)$$

for some distribution $P_{S_1UVX_0X_1X_2} = P_{X_{00}}P_{S_1}P_{UVX_0|S_1X_{00}}P_{X_1}P_{X_2}$, where U and V are auxiliary random variables that satisfy

$$I(U; Y_1) \geq I(S_1X_{00}; U) \quad (26a)$$

$$I(V; Y_2) \geq I(V; US_1X_{00}) \quad (26b)$$

Following from the above achievable region, we obtain an achievable region for the Gaussian channel by setting an appropriate joint distribution.

Proposition 8. *For Gaussian channel of model II, an inner bound on the capacity region*

for the regime when $Q_1 \rightarrow \infty$ consists of rate tuples (R_0, R_1, R_2) satisfying:

$$R_0 \leq \frac{1}{2} \log \left(1 + \frac{P_{00}}{P_{01} + P_{02} + 1} \right) \quad (27a)$$

$$R_1 \leq \frac{1}{2} \log \left(1 + \frac{P_1}{(1 - \frac{1}{\alpha})^2 P_{01} + P_{02} + 1} \right) \quad (27b)$$

$$R_2 \leq \frac{1}{2} \log \left(1 + \frac{P_2}{1 + \frac{(\beta-1)^2 P_{02}(P_{00}+P_{01})}{P_{02} + \beta^2(P_{00}+P_{01})}} \right) \quad (27c)$$

where $P_{00} + P_{01} + P_{02} \leq P_0$, $P_{00}, P_{01}, P_{02} \geq 0$, $0 \leq \alpha \leq \frac{2P_{01}}{1+P_{01}+P_{02}+P_1}$, and $P_{02}^2 + 2\beta P_{02}(P_{01} + P_{00}) \geq \beta^2(P_{00} + P_{01})(P_{02} + P_2 + 1)$.

Proof. The achievability follows from Proposition 7 by choosing jointly Gaussian distribution for random variables as follows:

$$\begin{aligned} U &= X_{01} + \alpha(S_1 + X_{00}), \quad V = X_{02} + \beta(X_{00} + X_{01}) \\ X_{00} &\sim \mathcal{N}(0, P_{00}), \quad X_{01} \sim \mathcal{N}(0, P_{01}), \quad X_{02} \sim \mathcal{N}(0, P_{02}) \\ X_1 &\sim \mathcal{N}(0, P_1), \quad X_2 \sim \mathcal{N}(0, P_2) \end{aligned}$$

where $X_{00}, X_{01}, X_{02}, X_1, X_2$ and S_1 are independent. □

By comparing the inner and outer bound, we reach similar conclusion as in Theorem 4.

Theorem 5. For the Gaussian channel of model II, in the regime with $Q_1 \rightarrow \infty$, for certain rate R_0 , i.e., certain P_{00} , if $\frac{1}{2}(1 + P_1 + P_0 - P_{00}) > \frac{P_0^2}{P_0+P_2+1} - P_{00}$, the line A-B is on the boundary of the capacity region, for

$$\text{Point A : } \left(\frac{1}{2} \log \left(1 + \frac{P_{00}}{P_0 - P_{00} + 1} \right), 0, \frac{1}{2} \log(1 + P_2) \right)$$

$$\text{Point B : } \left(\frac{1}{2} \log \left(1 + \frac{P_{00}}{P_0 - P_{00} + 1} \right), \frac{1}{2} \log \left(1 + \frac{P_1}{\frac{(1+P_0-P_{00}+P_1)^2(1+P_0+P_2)}{4(P_0^2-P_{00}(P_0+P_2+1))} - P_1} \right), \frac{1}{2} \log(1 + P_2) \right)$$

If $\frac{1}{2}(1 + P_1 + P_0 - P_{00}) \leq \frac{P_0^2}{P_0+P_2+1} - P_{00}$, the line A-B is characterized as

$$\text{Point A : } \left(\frac{1}{2} \log \left(1 + \frac{P_{00}}{P_0 - P_{00} + 1} \right), 0, \frac{1}{2} \log(1 + P_2) \right)$$

$$\text{Point B : } \left(\frac{1}{2} \log \left(1 + \frac{P_{00}}{P_0 - P_{00} + 1} \right), \frac{1}{2} \log \left(1 + \frac{P_1(P_0 + P_2 + 1)}{P_0 + (P_0 + 1)(P_2 + 1)} \right), \frac{1}{2} \log(1 + P_2) \right)$$

Furthermore, if $P_1 > P_0 - P_{00} + 1$, the line C-D is also on the boundary of the capacity region for the points

$$\text{Point C : } \left(\frac{1}{2} \log \left(1 + \frac{P_{00}}{P_0 - P_{00} + 1} \right), \frac{1}{2} \log(1 + P_0 - P_{00}), \frac{1}{2} \log \left(1 + \frac{P_2}{P_0 + 1} \right) \right)$$

$$\text{Point D : } \left(\frac{1}{2} \log \left(1 + \frac{P_{00}}{P_0 - P_{00} + 1} \right), \frac{1}{2} \log(1 + P_0 - P_{00}), 0 \right)$$

If $P_1 \leq P_0 - P_{00} - 1$, the line C - D is characterized as

$$\begin{aligned} \text{Point } C &: \left(\frac{1}{2} \log \left(1 + \frac{P_{00}}{P_0 - P_{00} + 1} \right), \frac{1}{2} \log(1 + P_1), \frac{1}{2} \log \left(1 + \frac{P_2}{P_1 + 2} \right) \right) \\ \text{Point } D &: \left(\frac{1}{2} \log \left(1 + \frac{P_{00}}{P_0 - P_{00} + 1} \right), \frac{1}{2} \log(1 + P_1), 0 \right) \end{aligned}$$

5 Model III: General K

In this section, we consider the Gaussian channel of model III with $K \geq 2$, in which there are multiple receivers with each interfered by an independent state. We first study the scenario, in which the helper dedicates to help two users without transmitting its own message, i.e., $W_0 = \phi$. We then extend the result to the more general scenario, in which the helper also has its own message destined for the corresponding receiver in addition to helping the two users, i.e., $W_0 \neq \phi$.

5.1 Scenario with Dedicated Helper ($W_0 = \phi$)

In this subsection, we study the scenario in which there are multiple receivers with each corrupted by an independently distributed state sequence, and the common helper (without transmission of its own messages) fully assists to cancel such state interference for all receivers. We note that this model is more general than model I, but does not include model II as a special case, because model II has one receiver that is not corrupted by state, but each receiver (excluding the helper) in model III is corrupted by an infinitely powered state sequence. Hence for model III, the challenge lies in the fact that the helper needs to assist multiple receivers to cancel interference caused by independent states. In this subsection, we first derive an outer bound on the capacity region, and then derive an inner bound based on a time-sharing scheme for the helper. Somewhat interestingly, comparing the inner and outer bounds concludes that the time-sharing scheme achieves the sum capacity under certain channel parameters, and we hence characterize certain segments of the boundary of the capacity region corresponding to the sum capacity under these channel parameters.

In the following, we first study the simple case with $K = 2$, which is more instructional. We then generalize our results to the case with $K \geq 2$. We first derive the outer bound on the capacity region.

Proposition 9. *For the Gaussian channel of model III with $W_0 = \phi$ and $K = 2$, an outer bound on the capacity region for the regime when $Q_1, Q_2 \rightarrow \infty$ consists of rate pairs (R_1, R_2)*

satisfying:

$$R_1 \leq \frac{1}{2} \log(1 + P_1) \quad (32)$$

$$R_2 \leq \frac{1}{2} \log(1 + P_2) \quad (33)$$

$$R_1 + R_2 \leq \frac{1}{2} \log(1 + P_0) \quad (34)$$

Proof. The proof is detailed in Appendix E. \square

We note that although the two transmitters transmit over a parallel channel, the above outer bound suggests that their sum rate is still subject to a certain constraint determined by the helper's power. This implies that it is not possible for one common helper to cancel the two independent high-power states simultaneously (i.e., using the common resource). This fact also suggests that a time-sharing scheme, in which the helper alternatively assists each receiver, can be desirable to achieve the sum rate upper bound (i.e., to achieve the sum capacity).

We hence design the following time-sharing achievable scheme. The helper splits its transmission duration into two time slots with the fraction γ of the total time duration for assisting receiver 1 and the fraction $1 - \gamma$ for assisting receiver 2. Each transmitter transmits only during the time slot that it is assisted by the helper, and keeps silent while the helper assisting the other transmitter. We note that the power constraints for transmitters 1 and 2 in their corresponding transmission time slots are $\frac{P_1}{\gamma}$ and $\frac{P_2}{1-\gamma}$, respectively.

Now at each transmission slot, the channel consists of one transceiver pair with the receiver corrupted by a high-power state, and one helper that assists the receiver to cancel the state interference. Such a model is a special case of the model studied in Section 3 (with the state-informed transmitter not having its own message). We rewrite the achievable rate for the model in Section 3 with power constraints P and P_0 respectively at the transmitter and the helper:

$$R(P, P_0) := \begin{cases} \frac{1}{2} \log(1 + P_0), & P \geq P_0 + 1 \\ \frac{1}{2} \log\left(1 + \frac{4P_0P}{4P_0 + (P_0 - P - 1)^2}\right), & P_0 - 1 \leq P \leq P_0 + 1 \\ \frac{1}{2} \log(1 + P), & P \leq P_0 - 1 \end{cases} \quad (35)$$

By employing the time-sharing scheme between the helper assisting one receiver and the other alternatively, we obtain the following achievable region.

Proposition 10. *For the Gaussian channel of model III with $W_0 = \phi$ and $K = 2$, in the regime with $Q_1, Q_2 \rightarrow \infty$, an inner bound on the capacity region consists of rate pairs*

(R_1, R_2) satisfying:

$$R_1 \leq \gamma R\left(\frac{P_1}{\gamma}, P_0\right) \quad (36a)$$

$$R_2 \leq (1 - \gamma)R\left(\frac{P_2}{1 - \gamma}, P_0\right) \quad (36b)$$

where $0 \leq \gamma \leq 1$ is the time-sharing coefficient.

We note that following from (35), the best possible single-user rate is $\frac{1}{2} \log(1 + P_0)$, which can be achieved if $P \geq P_0 + 1$. This best rate may not be possible if P is not large enough. Interestingly, in a time-sharing scheme, both transmitters can simultaneously achieve the best single user rate $\frac{1}{2} \log(1 + P_0)$ over their transmission fraction of time, because both of their powers get boosted over a certain fraction of time, although neither power is larger than $P_0 + 1$. In this way, the sum rate upper bound (34) can be achieved. The following theorem characterizes the sum capacity of the channel for the scenario describe above.

Theorem 6. *For the Gaussian channel of model III with $K = 2$ and $W_0 = \phi$, in the regime with $Q_1, Q_2 \rightarrow \infty$, if $P_1 + P_2 \geq P_0 + 1$, the sum capacity equals $\frac{1}{2} \log(1 + P_0)$. The rate points that achieve the sum capacity on the boundary of the capacity region are characterized as $(R_1, R_2) = \left(\gamma R\left(\frac{P_1}{\gamma}, P_0\right), (1 - \gamma)R\left(\frac{P_2}{1 - \gamma}, P_0\right)\right)$ for $\gamma \in \left(\max\left(1 - \frac{P_2}{P_0 + 1}, 0\right), \min\left(\frac{P_1}{P_0 + 1}, 1\right)\right)$.*

Proof. The proof is detailed in Appendix F. □

The above theorem implies the following characterization of the full capacity region under certain parameters.

Corollary 4. *For the Gaussian channel of model III with $W_0 = \phi$, in the regime with $Q_1, Q_2 \rightarrow \infty$, if $P_1, P_2 \geq P_0 + 1$, then the capacity region consists of the rate pair (R_1, R_2) satisfying $R_1 + R_2 \leq \frac{1}{2} \log(1 + P_0)$.*

We next provide channel examples to understand our inner and outer bounds, and in particular, Theorem 6 on the sum capacity. It can be seen that the power constraints fall into four cases, among which we consider the following three cases: case 1. $P_1 \geq P_0, P_2 \geq P_0$; case 2. $P_1 \geq P_0, P_2 < P_0$; and case 3. $P_1 < P_0, P_2 < P_0$ by noting that case 4 is opposite to case 2 and is omitted due to symmetry of the two transmitters.

- Case 1: $P_1 \geq P_0, P_2 \geq P_0$

We consider an example channel with $P_0 = 1, P_1 = 1.8$ and $P_2 = 1.5$. Figure 6 (a) plots the inner and outer bounds on the capacity region. In particular, the two bounds meet over the line segment B-C, which corresponds to the rate points $(R_1, R_2) = \left(\gamma R\left(\frac{P_1}{\gamma}, P_0\right), (1 - \gamma)R\left(\frac{P_2}{1 - \gamma}, P_0\right)\right)$ for $\gamma \in \left(\max\left(1 - \frac{P_2}{P_0 + 1}, 0\right), \min\left(\frac{P_1}{P_0 + 1}, 1\right)\right)$ as characterized in Theorem 6. All these rate points achieve the sum capacity. It can also be seen that although neither transmitter achieves the best possible single-user

rate, the sum capacity can be achieved due to the time-sharing scheme. We also note that, in this case, if the conditions in Corollary 4 are satisfied, the full capacity region is characterized.

- Case 2: $P_1 > P_0, P_2 \leq P_0$

We consider an example channel with $P_0 = 2, P_1 = 2.5$ and $P_2 = 0.8$. Figure 6 (b) plots the inner and outer bounds on the capacity region. Similarly to case 1, the two bounds meet over the line segment B-C as characterized in Theorem 6, and the points on such a line segment achieve the sum capacity. Differently from case 1, transmitter 2 achieves its point-to-point channel capacity indicated by the point A in Figure 6 (b). This is consistent with the single user rate provided in (35) for the case with $P_2 \leq P_0 - 1$.

- Case 3: $P_1 < P_0, P_2 < P_0$

We consider an example channel with $P_0 = 4, P_1 = 3$ and $P_2 = 3$. Figure 6 (c) plots the inner and outer bounds on the capacity region. The points on the line segment B-C achieve the sum capacity as characterized in Theorem 6, and the points A and D respectively achieve the point-to-point capacity for two transceiver pairs. This is consistent with the single-user rate provided in (35) for the case with $P_1, P_2 \leq P_0 - 1$.

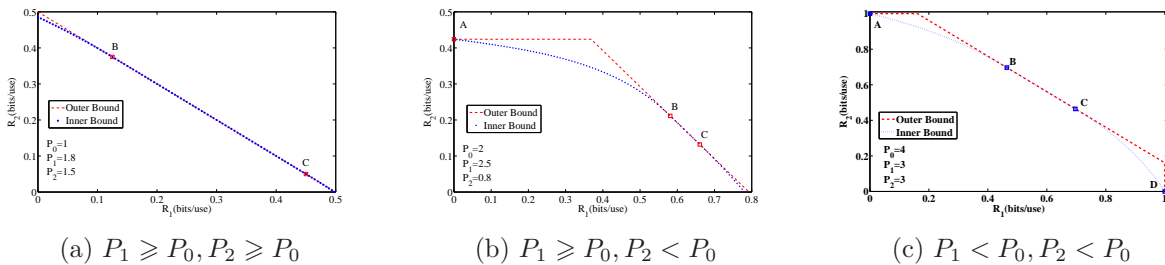


Figure 6: An illustration of the partial capacity boundary for the Gaussian channel of model III

We next generalize our results to the case with $K \geq 2$.

5.2 Extension: $W_0 \neq \phi$

In this subsection, we study the scenario, in which the helper also has its own message to transmit in addition to assisting the state-corrupted receivers, i.e., $W_0 \neq \phi$. The results we present below extend those in the preceding subsection for the scenario with $W_0 = \phi$ as well as the results in Section 3 for model I. The proof techniques combine those in Sections 5.1 and 3, and hence are omitted.

We first provide the outer bound as follows.

Proposition 11. For the Gaussian channel of model III, an outer bound on the capacity region for the regime when $Q_1, \dots, Q_K \rightarrow \infty$ consists of rate tuples (R_0, \dots, R_K) satisfying:

$$\begin{aligned} \sum_{k=0}^K R_k &\leq \frac{1}{2} \log(1 + P_0) \\ R_k &\leq \frac{1}{2} \log(1 + P_k) \quad k = 0, \dots, K \end{aligned}$$

Proof. The proof is detailed in Appendix G □

By utilizing time-sharing scheme, receivers share the helper's assistance with activating time portion γ_k for receiver k. The corresponding achievable region is as follows.

Proposition 12. For the Gaussian channel of model III, an inner bound consists of rate tuples (R_0, \dots, R_K) satisfying:

$$\begin{aligned} R_0 &\leq \sum_{k=1}^K \gamma_k \frac{1}{2} \log \left(1 + \frac{\bar{\beta}_k P_0}{\beta_k P_0 + 1} \right) \\ R_k &\leq \gamma_k R \left(\frac{P_k}{\gamma_k}, \beta_k P_0 \right) \\ \sum_{k=1}^K \gamma_k &= 1, \quad \gamma_k \geq 0 \quad \bar{\beta}_k + \beta_k = 1 \quad k = 1, \dots, K \end{aligned}$$

where $R(\cdot, \cdot)$ are the function defined in (35).

By comparing the inner and outer bound, we introduce a natural generalization of Theorem 6.

Theorem 7. The sum capacity of the Gaussian channel of model III is $\sum_{k=0}^K R_k \leq \frac{1}{2} \log(1 + P_0)$. The points on the boundary are characterized as

$$\begin{aligned} R_0 &\leq \sum_{k=1}^K \gamma_k \frac{1}{2} \log \left(1 + \frac{\bar{\beta}_k P_0}{\beta_k P_0 + 1} \right) \\ R_k &\leq \gamma_k \frac{1}{2} \log(1 + \beta_k P_0) \quad k = 1, \dots, K \end{aligned}$$

where

$$\begin{aligned} \sum_{k=1}^K \gamma_k &= 1, \quad \gamma_k \geq 0 \\ \frac{P_k}{\gamma_k} &\geq \beta_k P_0 + 1 \\ \bar{\beta}_k + \beta_k &= 1 \quad k = 1, \dots, K. \end{aligned}$$

6 Conclusion

In this paper, we proposed and studied parallel communication networks with a state-cognitive helper. We considered three models, and derived inner and outer bounds for each model. By comparing these bounds, we characterized full or certain segments of the boundary of the capacity region for some channel parameters. As we mentioned in Section 1, large state interference and state-cognitive helpers are well justified in practical wireless networks, and hence the achievable schemes developed here are promising to greatly improve the throughput of wireless networks. We also anticipate that the techniques that we develop in this paper will be helpful for studying various other multi-user state-dependent models with state-cognitive helpers.

Appendix

A Proof of Lemma 1

We use random codes and fix the following joint distribution:

$$P_{S_1 X'_0 U X_0 X_1 Y_0 Y_1} = P_{S_1} P_{X'_0} P_{U|S_1 X'_0} P_{X_0|U S_1 X'_0} P_{X_1} P_{Y_0|X_0} P_{Y_1|X_0 X_1 S_1}.$$

Let $T_\epsilon^n(P_{S_1 X'_0 U X_0 X_1 Y_0 Y_1})$ denote the strongly joint ϵ -typical set based on the above distribution. For a given sequence x^n , let $T_\epsilon^n(P_{U|X}|x^n)$ denote the set of sequences u^n such that (u^n, x^n) is jointly typical based on the distribution P_{XU} .

1. Codebook Generation

- Generate $2^{n\tilde{R}}$ codewords $u^n(v)$ with the probability of P_U , in which $v \in [1, 2^{n\tilde{R}}]$.
- Generate 2^{nR_0} codewords $x'_0{}^n(w_0)$ with the probability of $P_{X'_0}$, in which $w_0 \in [1, 2^{nR_0}]$.
- Generate 2^{nR_1} codewords $x_1^n(w_1)$ with the probability of P_{X_1} , in which $w_1 \in [1, 2^{nR_1}]$.

2. Encoding

- Encoder 0: Given w_0 , map w_0 into $x'_0{}^n(w_0)$. For each $x'_0{}^n(w_0)$, select \tilde{v} such that $(u^n(\tilde{v}), s_1^n, x'_0{}^n(w_0)) \in T_\epsilon^n(P_{S_1} P_{X'_0} P_{U|S_1 X'_0})$. If $u^n(\tilde{v})$ cannot be found, set $\tilde{v} = 1$. Then map $(s_1^n, u^n(\tilde{v}), x'_0{}^n(w_0))$ into $x_0^n = f^{(n)}(x'_0{}^n(w_0), s_1^n, u^n(\tilde{v}))$. It can be shown that such $u^n(\tilde{v})$ exists with high probability for large n if

$$\tilde{R} > I(U; S_1 X'_0). \tag{37}$$

- Encoder 1: Given w_1 , map w_1 into $x_1^n(w_1)$.

3. Decoding

- Decoder 0: Given y_0^n , find \hat{w}_0 such that $(x_0'^n(\hat{w}_0), y_0^n) \in T_\epsilon^n(P_{X_0'Y_0})$. If no or more than one \hat{w}_0 can be found, declare error. It can be shown that the decoding error is small for sufficient large n if

$$R_0 \leq I(X_0'; Y_0). \quad (38)$$

- Decoder 1: Given y_1^n , find a pair (\hat{v}, \hat{w}_1) such that $(u^n(\hat{v}), x_1^n(\hat{w}_1), y_1^n) \in T_\epsilon^n(P_{UX_1Y_1})$. If no or more than one such pair can be found, then declare error. It can be shown that decoding is successful with small probability of error for sufficiently large n if the following conditions are satisfied

$$R_1 \leq I(X_1; Y_1|U), \quad (39)$$

$$\tilde{R} \leq I(U; Y_1|X_1), \quad (40)$$

$$R_1 + \tilde{R} \leq I(UX_1; Y_1). \quad (41)$$

We note that (40) corresponds to the decoding error for the index v , which is not the message of interest. Hence, the bound (40) can be removed. Hence, combining (37), (38), (39), and (41) and eliminating \tilde{R} , and we obtain the desired achievable region as in Lemma 1.

B Proof of Proposition 2

We first bound the single-user rate R as follows.

$$R_1 \leq \frac{1}{2} \log(1 + P_1) \quad (42)$$

We then bound the sum rate as follows. For the message W_0 , based on Fano's inequality, we have

$$\begin{aligned} nR_0 &\leq I(W_0; Y_0^n) + n\epsilon_n \\ &= h(Y_0^n) - h(Y_0^n|W_0) + n\epsilon_n, \end{aligned} \quad (43)$$

where $\epsilon_n \rightarrow 0$ as $n \rightarrow \infty$.

For the message W_1 , based on Fano's inequality, we have

$$\begin{aligned} nR_1 &\leq I(W_1; Y_1^n) + n\epsilon_n \\ &= h(Y_1^n) - h(Y_1^n|W_1) + n\epsilon_n \\ &\leq h(Y_1^n) - h(Y_1^n|W_1X_1^n) + n\epsilon_n \\ &= h(Y_1^n) - h(X_0^n + S_1^n + N_1^n) + n\epsilon_n \\ &\leq h(Y_1^n) - h(X_0^n + S_1^n + N_1^n|W_0Y_0^n) + n\epsilon_n \end{aligned} \quad (44)$$

Summation of (43) and (44) yields

$$n(R_0 + R_1) \leq h(Y_0^n) + h(Y_1^n) - h(Y_0^n, X_0^n + S_1^n + N_1^n | W_0) \quad (45)$$

Since the capacity region of the channel depends on only marginal distributions of (X_0, Y_0) and (X_0, X_1, S, Y_1) , setting $N_1 = N_0$ does not change the capacity region. Thus,

$$\begin{aligned} n(R_0 + R_1) &\leq h(Y_0^n) + h(Y_1^n) - h(S_1^n, X_0^n + N_1^n | W_0) \\ &\leq h(Y_0^n) + h(Y_1^n) - h(S_1^n) - h(N_1^n) \\ &\leq \frac{n}{2} \log(1 + P_0) + \frac{n}{2} \log \left(1 + \frac{P_0 + P_1 + 1}{Q_1} \right) \end{aligned} \quad (46)$$

As $Q_1 \rightarrow \infty$, the second term of the above bound goes to 0, and we have

$$n(R_0 + R_1) \leq \frac{1}{2} \log(1 + P_0). \quad (47)$$

C Proof of Proposition 3

The single rate bound is based on the result in Section 3 for model I and the point-to-point channel capacity.

For the sum rate bound, according to Fano's inequality, we have

$$\begin{aligned} n(R_1 + R_2) &\leq I(W_1; Y_1^n) + I(W_2; Y_2^n) \\ &= h(Y_1^n) - h(Y_1^n | W_1) + h(Y_2^n) - h(Y_2^n | W_2) \\ &\stackrel{(a)}{=} h(Y_1^n) - h(Y_1^n | W_1 X_1^n) + h(Y_2^n) - h(Y_2^n | W_2 X_2^n) \\ &= h(Y_1^n) - h(X_0^n + S_1^n + N_1^n) + h(Y_2^n) - h(X_0^n + N_2^n) \\ &\leq h(Y_1^n) - h(X_0^n + S_1^n + N_1^n | X_0^n + N_1^n) \\ &\quad + h(Y_2^n) - h(X_0^n + N_2^n) \end{aligned}$$

where (a) follows from that X_1^n is function of W_1 , and X_2^n is function of W_2 , and they are independent from X_0^n , state and noise. Because the two decoders decode based on the marginal distribution only, setting $N_1^n = N_2^n$ does not influence the channel capacity,

therefore,

$$\begin{aligned}
n(R_1 + R_2) &\leq h(Y_1^n) - h(X_0^n + S_1^n + N_1^n, X_0^n + N_1^n) + h(Y_2^n) \\
&= h(Y_1^n) - h(S_1^n, X_0^n + N_1^n) + h(Y_2^n) \\
&= h(Y_1^n) - h(S_1^n) - h(X_0^n + N_1^n | S_1^n) + h(Y_2^n) \\
&\leq h(Y_1^n) - h(S_1^n) - h(X_0^n + N_1^n | S_1^n, X_0^n) + h(Y_2^n) \\
&\stackrel{(b)}{=} h(X_0^n + X_1^n + S_1^n + N_1^n) - h(S_1^n) + h(X_0^n + X_2^n + N_1^n) - h(N_1^n) \\
&\leq \frac{n}{2} \log 2\pi e (P_1 + P_0 + \sum_{i=1}^n E(X_{0i} S_{1i}) + Q + 1) - \frac{n}{2} \log(2\pi e Q) \\
&\quad + \frac{n}{2} \log 2\pi e (P_0 + P_2 + 1) - \frac{n}{2} \log(2\pi e) \\
&\leq \frac{n}{2} \log \left(\frac{P_1 + P_0 + 2\sqrt{P_0 Q_1} + Q_1 + 1}{Q_1} \right) + \frac{n}{2} \log(P_0 + P_2 + 1) \\
&\approx \frac{n}{2} \log(P_0 + P_2 + 1) \quad (Q_1 \rightarrow \infty)
\end{aligned}$$

where (b) follows from that X_0^n and S_1^n are independent from N_1^n .

D Proof of Proposition 4

We use random codes and fix the following joint distribution:

$$P_{S_1 U_1 V X_0 X_1 X_2 Y_1 Y_2} = P_{V U S_1} P_{X_0 | V U S_1} P_{X_1} P_{X_2} P_{Y_1 | X_0 X_1 S_1} P_{Y_2 | X_0 X_2}.$$

Let $T_\epsilon^n(P_{S_1 U V X_0 X_1 X_2 Y_1 Y_2})$ denote the strongly joint ϵ -typical set based on the above distribution. For a given sequence x^n , let $T_\epsilon^n(P_{U|X}|x^n)$ denote the set of sequences u^n such that (u^n, x^n) is jointly typical based on the distribution P_{XU} .

1. Codebook Generation

- Generate $2^{n\tilde{R}_1}$ codewords $u^n(t)$ with the probability of P_U , in which $t \in [1, 2^{n\tilde{R}_1}]$.
- Generate $2^{n\tilde{R}_2}$ codewords $v^n(k)$ with the probability of P_V , in which $k \in [1, 2^{n\tilde{R}_2}]$.
- Generate 2^{nR_1} codewords $x_1^n(w_1)$ with the probability of P_{X_1} , in which $w_1 \in [1, 2^{nR_1}]$.
- Generate 2^{nR_2} codewords $x_2^n(w_2)$ with the probability of P_{X_2} , in which $w_2 \in [1, 2^{nR_2}]$.

2. Encoding

- Encoder 0: Given s_1^n , find \tilde{t} , such that $(u^n(\tilde{t}), s_1^n) \in T_\epsilon^n(P_{S_1 U})$. Such $u^n(\tilde{t})$ exists with high probability for large n if

$$\tilde{R}_1 \geq I(S_1; U). \quad (48)$$

- For each \tilde{t} selected, select \tilde{k} , such that $(v^n(\tilde{k}), u^n(\tilde{t}), s_1^n) \in T_\epsilon^n(P_{VUS_1})$. Such $v^n(\tilde{k})$ exists with high probability for large n if

$$\tilde{R}_2 \geq I(S_1U; V). \quad (49)$$

- Map (s_1^n, u^n, v^n) into x_0^n
- Encoder 1 and 2: Map w_1 into x_1^n , and map w_2 into x_2^n .

3. Decoding

- Decoder 1: Given y_1^n , find (\hat{w}_1) such that $(x_1^n(\hat{w}_1), u^n(\hat{t}), y_1^n) \in T_\epsilon^n(P_{X_1UY_1})$. If no or more than one \hat{w}_1 can be found, declare an error. One can show that the decoding error is small for sufficient large n if

$$R_1 \leq I(X_1; Y_1U) \quad (50)$$

$$R_1 + \tilde{R}_1 \leq I(X_1U; Y_1) \quad (51)$$

- Decoder 2: Given y_2^n , find (\hat{w}_2) such that $(x_2^n(\hat{w}_2), v^n(\hat{k}), y_2^n) \in T_\epsilon^n(P_{X_2VY_2})$. If no or more than one \hat{w}_2 can be found, declare an error. One can show that the decoding error is small for sufficient large n if

$$R_2 \leq I(X_2; Y_2V) \quad (52)$$

$$R_2 + \tilde{R}_2 \leq I(X_2V; Y_2) \quad (53)$$

According to (48)-(53), exploit the Foriour-Mozkin elimination to eliminate \tilde{R}_1 and \tilde{R}_2 , and we have the desired achievable region.

E Proof of Proposition 9

First of all, R_1 and R_2 is bounded by single rate bound respectively.

For the sum rate bound, we start from the Fano's inequality

$$\begin{aligned} n(R_1 + R_2) &\leq I(W_1; Y_1^n) + I(W_2; Y_2^n) \\ &= h(Y_1^n) - h(Y_1^n|W_1) + h(Y_2^n) - h(Y_2^n|W_2) \\ &\stackrel{(a)}{=} h(Y_1^n) - h(Y_1^n|W_1X_1^n) + h(Y_2^n) - h(Y_2^n|W_2X_2^n) \\ &= h(Y_1^n) - h(X_0^n + S_1^n + N_1^n) + h(Y_2^n) - h(X_0^n + S_2^n + N_2^n) \\ &\leq h(Y_1^n) - h(X_0^n + S_1^n + N_1^n|X_0^n + N_1^n) \\ &\quad + h(Y_2^n) - h(X_0^n + S_2^n + N_2^n|X_0^n + N_2^n, X_0^n + S_1^n + N_1^n) \\ &\quad + h(X_0^n + N_1^n) - h(X_0^n + N_1^n) \end{aligned}$$

where (a) follows from that X_1^n is function of W_1 , and X_2^n is function of W_2 , and they are independent from X_0^n , state and noise. Because the two decoders decode based on

the marginal distribution only, setting $N_1^n = N_2^n$ does not influence the channel capacity, therefore,

$$\begin{aligned}
n(R_1 + R_2) &\leq h(Y_1^n) - h(X_0^n + S_1^n + N_1^n, X_0^n + S_2^n + N_1^n, X_0^n + N_1^n) \\
&\quad + h(Y_2^n) + h(X_0^n + N_1^n) \\
&= h(Y_1^n) - h(S_1^n, S_2^n, X_0^n + N_1^n) + h(Y_2^n) + h(X_0^n + N_1^n) \\
&= h(Y_1^n) - h(S_1^n) - h(S_2^n) - h(X_0^n + N_1^n | S_1^n, S_2^n) \\
&\quad + h(Y_2^n) + h(X_0^n + N_1^n) \\
&\leq h(Y_1^n) - h(S_1^n) - h(S_2^n) - h(X_0^n + N_1^n | S_1^n, S_2^n, X_0^n) \\
&\quad + h(Y_2^n) + h(X_0^n + N_1^n) \\
&\stackrel{(b)}{=} h(X_0^n + X_1^n + S_1^n + N_1^n) - h(S_1^n) - h(S_2^n) - h(N_1^n) \\
&\quad + h(X_0^n + X_2^n + S_2^n + N_1^n) + h(X_0^n + N_1^n) \\
&\leq \frac{n}{2} \log 2\pi e (P_1 + P_0 + \sum_{i=1}^n E(X_{0i} S_{1i}) + Q_1 + 1) - \frac{n}{2} \log(2\pi e Q_1) - \frac{n}{2} \log(2\pi e Q_2) \\
&\quad - \frac{n}{2} \log(2\pi e) + \frac{n}{2} \log 2\pi e (P_2 + P_0 + \sum_{i=1}^n E(X_{0i} S_{2i}) + Q_2 + 1) + \frac{n}{2} \log 2\pi e (P_0 + 1) \\
&\leq \frac{n}{2} \log \left(\frac{P_1 + P_0 + 2\sqrt{P_0 Q_1} + Q_1 + 1}{Q_1} \right) + \frac{n}{2} \log \left(\frac{P_2 + P_0 + 2\sqrt{P_0 Q_2} + Q_2 + 1}{Q_2} \right) \\
&\quad + \frac{n}{2} \log(P_0 + 1) \\
&\approx \frac{n}{2} \log(P_0 + 1) \quad (Q_1 \rightarrow \infty, Q_2 \rightarrow \infty)
\end{aligned}$$

where (b) follows from that X_0^n , S_1^n and S_2^n are independent from N_1^n .

F Proof of Proposition 6

The theorem can be proved in two parts, 1. if $P_1 + P_2 \geq P_0 + 1$, the sum capacity is obtained; 2. characterize γ such that for this time allocation, the rate achieved is on the capacity boundary.

1. For a certain P_0 , we consider the following two cases.

a). If the power constraint satisfies $P_1 + P_2 = P_0 + 1$, by following Proposition 10, setting $\gamma = \frac{P_1}{P_1 + P_2}$, the point $(R_1, R_2) = (\frac{P_1}{2(P_1 + P_2)} \log(1 + P_0), \frac{P_2}{2(P_1 + P_2)} \log(1 + P_0))$ is achieved, which is also on the outer bound in Proposition 9.

b). If $P_1 + P_2 \geq P_0 + 1$, the outer bound does not change for the same P_0 , we use the power $\tilde{P}_1 + \tilde{P}_2 = P_0 + 1$ and $\tilde{P}_1 \leq P_1$, $\tilde{P}_2 \leq P_2$ for each transmitter and obtain the sum capacity as concluded in a).

2. We start from considering the constraint for γ under which the rate pair (R_1, R_2)

achieves the sum capacity, i.e.

$$\frac{P_2}{1-\gamma} \geq P_0 + 1 \quad (54)$$

$$\frac{P_1}{\gamma} \geq P_0 + 1. \quad (55)$$

It is clear that (54) implies

$$\gamma \geq 1 - \frac{P_2}{P_0 + 1},$$

and (55) implies

$$\gamma \leq \frac{P_1}{P_0 + 1}.$$

Considering $0 \leq \gamma \leq 1$, we obtain the desired bounds on γ .

G Proof of Proposition 11

The individual rate is first bounded by the point-to-point channel capacity, respectively.

We then bound the sum rate. By following the Fano's inequality, we have

$$\begin{aligned} \sum_{k=0}^K nR_k &\leq \sum_{k=0}^K I(W_k; Y_k^n) \\ &= \sum_{k=0}^K [h(Y_k^n) - h(Y_k^n | W_k)] \\ &\stackrel{(a)}{=} h(Y_0^n) - h(X_0^n + N_0^n | W_0) + \sum_{k=1}^K [h(Y_k^n) - h(Y_k^n | W_k X_k^n)] \\ &= h(Y_0^n) - h(X_0^n + N_0^n | W_0) + \sum_{k=1}^K [h(Y_k^n) - h(X_0^n + S_k^n + N_k^n)] \\ &\leq h(Y_0^n) - h(X_0^n + N_0^n | W_0) \\ &\quad + \sum_{k=1}^K [h(Y_k^n) - h(X_0^n + S_k^n + N_k^n | X_0^n + N_0^n, W_0, X_0^n + S_{k-1}^n + N_{k-1}^n, \dots, X_0^n + S_1^n + N_1^n)] \\ &= [\sum_{k=0}^K h(Y_k^n)] - h(X_0^n + S_1^n + N_1^n, \dots, X_0^n + S_K^n + N_K^n, X_0^n + N_0^n | W_0) \end{aligned}$$

where (a) follows from that X_k^n is function of W_k , and they are independent from X_0^n , state and noise. Because the decoders decode based on the marginal distribution only, we can set

$N_k^n = N_0^n$ for $k = 1, \dots, K$, therefore,

$$\begin{aligned}
\sum_{k=1}^K nR_k &\leq \left[\sum_{k=0}^K h(Y_k^n) \right] - h(S_1^n, \dots, S_K^n, X_0^n + N_0^n) \\
&= h(Y_0^n) + \sum_{k=1}^K [h(Y_k^n) - h(S_k^n)] - h(X_0^n + N_0^n | S_1^n, \dots, S_K^n, W_0) \\
&\leq h(X_0^n + N_0^n) + \sum_{k=1}^K [h(Y_k^n) - h(S_k^n)] - h(X_0^n + N_0^n | S_1^n, \dots, S_K^n, W_0, X_0^n) \\
&\stackrel{(b)}{=} h(X_0^n + N_0^n) + \sum_{k=1}^K [h(X_0^n + X_k^n + S_k^n + N_k^n) - h(S_k^n)] - h(N_0^n) \\
&\leq \sum_{k=1}^K \left[\frac{n}{2} \log 2\pi e (P_k + P_0 + \sum_{i=1}^n E(X_{0i} S_{ki}) + Q_k + 1) - \frac{n}{2} \log(2\pi e Q_k) \right] \\
&\quad - \frac{n}{2} \log(2\pi e) + \frac{n}{2} \log 2\pi e (P_0 + 1) \\
&\leq \sum_{k=1}^K \frac{n}{2} \log \left(\frac{P_k + P_0 + 2\sqrt{P_0 Q_k} + Q_k + 1}{Q_k} \right) + \frac{n}{2} \log(P_0 + 1) \\
&\rightarrow \frac{n}{2} \log(P_0 + 1) \text{ as } Q_k \rightarrow \infty
\end{aligned}$$

References

- [1] Y. Steinberg, "Correspondence coding for the degraded broadcast channel with random parameters, with causal and noncausal side information," *IEEE Trans. Inform. Theory*, vol. 51, no. 8, August 2005.
- [2] A. Lapidoth and L. Wang, "The state-dependent semideterministic broadcast channel," *IEEE Trans. Inform. Theory*, vol. 59, no. 4, pp. 2242–2251, Apr. 2013.
- [3] A. Somekh-Baruch, S. Shamai (Shitz), and S. Verdú, "Cooperative multiple-access encoding with states available at one transmitter," *IEEE Trans. Inform. Theory*, vol. 54, no. 10, pp. 4448–4469, October 2008.
- [4] S. P. Kotagiri and J. N. Laneman, "Multiaccess channels with state known to some encoders and independent messages," *EURASIP Journal on Wireless Communications and Networking*, 2008.
- [5] I.-H. Wang, "Distributed interference cancellation in multiple access channels," *IEEE Trans. Inform. Theory*, vol. 58, no. 5, pp. 2781–2787, May 2012.
- [6] B. Akhbari, M. Mirmohseni, and M. R. Aref, "Compress-and-forward strategy for the relay channel with non-causal state information," in *Proc. IEEE Int. Symp. Information Theory (ISIT)*, Seoul, Korea, Jul. 2009.

- [7] A. Zaidi, S. Shamai (Shitz), P. Piantanida, and L. Vandendorpe, “Bounds on the capacity of the relay channel with noncausal state at source,” *IEEE Trans. Inform. Theory*, vol. 59, no. 5, pp. 2639–2672, May 2013.
- [8] L. Zhang, J. Jiang, and S. Cui, “Gaussian interference channel with state information,” *IEEE Trans. Wireless Commun.*, vol. 12, no. 8, pp. 4058–4071, Aug 2013.
- [9] R. Duan and Y. Liang, “Bounds and capacity theorems for cognitive interference channels with state,” *IEEE Trans. Inform. Theory*, submitted, June 2012; revised, October 2013.
- [10] R. Duan, Y. Liang, and S. Shamai (Shitz), “On the capacity region of gaussian interference channels with state,” in *Proc. IEEE Int. Symp. Information Theory (ISIT)*, Istanbul, Turkey, Jul. 2013.
- [11] S. Ghasemi-Goojani and H. Behroozi, “On the achievable rate-regions for state-dependent Gaussian interference channel,” Available at <http://arxiv.org/abs/1301.5535>, submitted in January 2013.
- [12] S. Gel’fand and M. Pinsker, “Coding for channels with random parameters,” *Probl. Contr. Inf. Theory*, vol. 9, no. 1, pp. 19–31, January 1980.
- [13] S. Hajizadeh, M. Monemizadeh, E. Bahmani, G. A. Hodtani, and M. Joneidi, “State-dependent z channel,” Available at <http://arxiv.org/abs/1301.6272>.
- [14] A. J. Khisti, U. Erez, A. Lapidoth, and G. Wornell, “Carbon copying onto dirty paper,” *IEEE Trans. Inform. Theory*, vol. 53, no. 5, pp. 1814–1827, May 2007.
- [15] M. H. M. Costa, “Writing on dirty paper,” *IEEE Trans. Inform. Theory*, vol. 29, no. 3, pp. 439–441, May 1983.
- [16] I. Sason, “On achievable rate regions for the Gaussian interference channel,” *IEEE Trans. Inform. Theory*, vol. 50, no. 6, pp. 1345–1356, June 2004.
- [17] P. Mitran, N. Devroye, and V. Tarokh, “On compound channels with side-information at the transmitter,” *IEEE Trans. Inform. Theory*, vol. 52, no. 4, pp. 1745–1755, April 2006.
- [18] P. Piantanida and S. Shamai (Shitz), “On the capacity of compound state-dependent channels with states known at the transmitter,” in *Proc. IEEE Int. Symp. Information Theory (ISIT)*, June 2010, pp. 624–628.